HW 1.7: 8(use contradiction), 22, 24, 26,30

**8.** Prove that if *n* is a perfect square, then *n* + 2 is not a perfect square.

**-Assume ~P (n is not a perfect square)**

**-n = m2**

**-There exists an integer k such that n+2 = k2**

**-if m = 0, then n + 2 = 0 + 2, which is not a perfect square.**

**-m therefore must be ≥ 1, or m + 1.**

**- (m + 1)2 = m2 + 2m + 1 = n + 2m + 1 ≥ n + 2(1) + 1 > n + 2.**

**Because when m ≥ 1 n cannot be a perfect square, we have a contradiction of the original hypothesis, so we can conclude the statement is true.**

**22.** Show that if you pick three socks from a drawer containing just blue socks and black socks, you must get either a pair of blue socks or a pair of black socks.

**-Use exhaustive proof**

**-Our possible options are:**

**(black, black, black): True, we would end up with a pair of black socks**

**(black, black, blue): True, we would end up with a pair of black socks**

**(black, blue, blue): True, we would end up with a pair of blue socks**

**(blue, blue, blue): True, we would end up with a pair of blue socks**

**Because in each case it is true that we will get either a pair of blue socks or a pair of black socks, we can conclude that the statement is true.**

**24.** Show that at least three of any 25 days chosen must fall in the same month of the year.

**-Show that the statement is false, or that ~P is true.**

**-Prove that 2 of any 25 days chosen must fall in the same month of the year.**

**-Because each year contains 12 months, there can only be a maximum of 24 days chosen for 2 days to fall within the same month of the year.**

**-Because this contradicts the hypothesis, we've shown that ~P is true, and we can conclude that p is true, and that for 3 of any 25 days chosen will fall in the same month.**

**26.** Prove that if *n* is a positive integer, then *n* is even if and only if 7*n* + 4 is even.

**-Show that the bi conditional statement is true by proving each condition.**

**P= ""n is even"**

**Q = "7n + 4 is even"**

**P → Q, Q → P**

**- n is even when an integer k exists such that n = 2k. 7(2k) + 4 = 14k + 4 = 2(7k + 2). If we make 7k + 2 equal to M, then n = 2(m), which is our definition of an even integer. We can thus conclude that P → Q is true.**

**-n is odd when an integer k exists such that n = 2k + 1. For 7n + 4 to be odd, n must be equal to 2k + 1. 7(2k + 4) + 1 = 14k + 5. Because 14k + 5 will always return an odd number, then if 7n + 4 is odd, then n will be odd. We can thus conclude that when 7n + 4 is even, n will be even (Q → P).**

**30.** Show that these three statements are equivalent, where *a* and *b* are real numbers: (*i*) *a* is less than *b*, (*ii*) the average of *a* and *b* is greater than *a*, and (*iii*) the average of *a* and *b* is less than *b*.

**P: A < B**

**Q: (A + B)/2 > A**

**R: (A + B)/2 < B**

**-Show that P → Q, P→ R, Q → R, and R → P to prove true**

**- If A < B, then B ≥ (A + 1). A + A + 1/2 will produce (2A + 1)/2 or A + .5 which is > A. Therefore, if B > A, then the average of a and b is greater than a. P → Q is true.**

**- If A < B, then A ≤ (B - 1). B + B -1/2 will produce (2B - 1)/2 or B - .5 which is < B. Therefore, if B > A, then the average of a and b is less than b. P → R is true.**

**-If (A + B)/2 > A is true, then (A + B)/2 < B is true. We showed in our P → Q proof that for the average of a and b to be greater than A, then B must be greater than A. We similarly showed for our P → R proof that for the average of a and b to be less than b that B must be less than A. Therefore, each can be true only if B > A. So if one is true, the other is as well. Q → R is true.**

**-Show ~R ( The average of A and B is greater than B) to prove that if the average of A and B is less than B, then A is less than B. (A+B/2 >B) For this to be true, then A ≥ (B + 1) where B + B + 1/2 returns B + .5 > B. By showing the contradiction is true, we can also conclude that the original conditional, R → P is true.**